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Revision Section

Revision / Consolidation

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Higher Still

Higher Maths

Formulae List

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle, centre $(-g, -f)$ and radius $\sqrt{(g^2 + f^2 - c)}$

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle, centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Trigonometric Formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - \sin^2 A \end{aligned}$$

$$\sin 2A = 2 \sin A \cos A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Higher

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Mathematics 1

Equations of Lines

1. Find the equation of the straight line which passes through the point (–2,6) and is :–
   (a) parallel to the line with equation $x = 3$.
   (b) perpendicular to the line with equation $y + 2x = 0$.
   (c) parallel to the line with equation $y - 3x = 4$.

2. Find the equation of the median PS of the triangle PQR where the coordinates of P, Q and R are :– (–2,3), (–3,–4) and (5,2) respectively.

3. Find the equation of the perpendicular bisector of the line joining M(2,–1) and N(8,3).

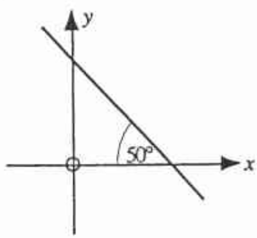
4. P(–3,5), Q(–1,–2) and R(5,1) are the vertices of a triangle PQR. Find the equation of PS, the altitude from P to QR.

5. K, L and M are the points (–5,–8), (12,–1) and (13,4) respectively.
   (a) Find the equation of KM.
   (b) If kite KLMN is completed, with KM the axis of symmetry, find the equation of LN and hence find the coordinates of the mid point of LN.

6. C has coordinates (4,7), D(–2,3) and E(1,9).
   (a) Find the equation of the line through the mid point of CD, parallel to DE.
   (b) Verify that this line passes through the mid point of CE.

7. Points G(0,–10), H(10,3) and I(–4,10) are vertices of ΔGHI. Find :–
   (a) the equations of the altitude GP and of median HQ of this triangle
   (b) the coordinates of the point of intersection of GP and HQ.

8. Find the gradient of this straight line. ⟶



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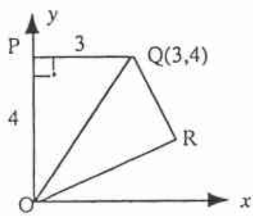


OPQR is a kite.
Calculate the gradient of line OR.
(correct to 2 decimal places)

Differentiation

10. Differentiate the following with respect to x :–

(a) $f(x) = 5x^4 - 3x - 17$.

(b) $y = 6x^2 - \dfrac{3}{x}$.

(c) $y = \dfrac{9}{x^2} + x\sqrt{x}$.

(d) $f(x) = 3\sqrt{x}(x-3)$.

(e) $f(x) = \dfrac{6x^2 - 8x + 5}{2x}$.

11. If $f(x) = \sqrt[4]{x} - \dfrac{1}{\sqrt[4]{x}}$, find $f'(16)$.

12. A ball is thrown vertically upwards.
The height H metres of the ball s seconds after it is thrown, is given by the formula :–

$$H = 36s - 6s^2$$

(a) Find the rate of change of height, with respect to the time of the ball just as it is thrown.

(b) Find the speed of the ball after 3 seconds and explain your answer.

13. (a) Find the equation of the tangent to the curve with equation

$y = 5x^3 - 8x^2$ at the point where $x = 1$

(b) Find the x coordinate of each of the points on the curve

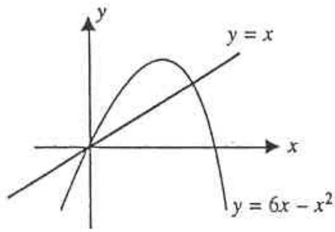
$$y = 2x^3 - 3x^2 - 12x + 12$$

at which the tangent is parallel to the x–axis.

14. Find the gradient of the tangent to the parabola

$$y = 6x - x^2 \text{ at } (0,0)$$

Hence calculate the size of the angle between the line $y = x$ and this tangent.



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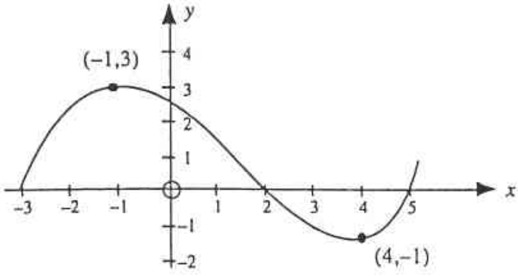


A sketch of a cubic function, $f(x)$, with domain $-3 \le x \le 5$, is shown.

Sketch the graph of the derived function $f'(x)$, for the same domain.

cont...

16. Find the maximum and minimum values of :–

$f(x) = 6x^2 - x^3$ in the closed interval $-2 \leq x \leq 2$.

17. Find the stationary values of the function defined by :–

$f(x) = 2 + 3x - x^3$

Hence, or otherwise, sketch the graph of $f(x)$ stating where the graph meets the axes.

18. (a) Given that $f(x) = \sqrt{x}(x-3)$, where only the positive value of $\sqrt{x}$ is taken for each value of $x > 0$, find $f'(x)$.

(b) State the coordinates of the point on the curve $y = f(x)$ where $x = 4$ and obtain the the equation of the tangent to the curve at this point.

(c) Show that $(1,-2)$ is a stationary point on the curve and sketch the curve for values of x in the interval $0 \leq x \leq 4$ indicating the intersection with the x and y axes.

Graphs/Functions

19. On a suitable set of real numbers, functions f and g are defined by :–

$$f(x) = \frac{1}{(x+3)} \text{ and } g(x) = \frac{1}{x} - 3$$

Find f(g(2)).

20. The functions h and k are defined by :–

h: $x \longrightarrow x^2$ and k: $x \longrightarrow x + 1$

Find in its simplest form :– h(k(p)) – k(h(p)).

21. $g(x) = x^2 - 1$; $x \varepsilon R$, defines a function g(x).

$h(x) = \frac{x-2}{x}$; $x \varepsilon R$, $x \neq 0$, defines a function h(x).

(a) Explain why g has no inverse function.

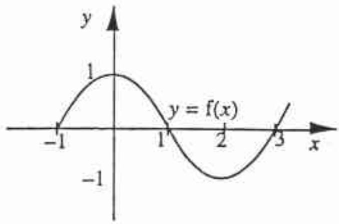
(b) Define the inverse function $h^{-1}(x)$, stating a suitable domain.

22. The graph of y = f(x) is shown.
On separate diagrams, sketch the graphs of :–

(a) $y = f(x) + 1$ (b) $y = f(x+1)$

(c) $y = -f(x) + 1$ (d) $y = -f(x+2)$

(e) $y = 2(f(x))$ (f) $y = 1 - f(x)$.



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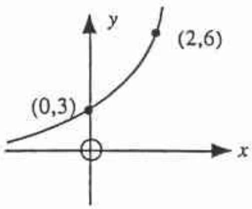
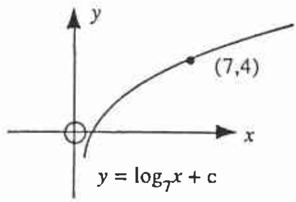


Part of the graph $y = a^x + b$ is shown.
Find the values of a and b.

24. The sketch shown opposite shows part of a logarithmic function.
Find the value of c.



25. Let p: $x \longrightarrow \sin x°$; q: $x \longrightarrow x^2$; r: $x \longrightarrow 1 - 2x$; be mappings on the set of Real Numbers R.

(a) Find a formula, in its simplest form, for the function s(x), such that s(x) = r(q(p(x))).
Hence, or otherwise, find the image of 30 under the mapping s.

(b) State, for each of p, q and r, whether there exists an inverse mapping on R and where it does exist, give the formula for it.

Completing the Square

26. Express each of the following in the form $a(x + p)^2 + q$ by completing the square.

 (a) $x^2 + 6x$ (b) $x^2 + 10x + 1$ (c) $5 + 4x - x^2$

 (d) $2x^2 + 6x + 1$ (e) $4 + 6x - 3x^2$.

27. In each of the above cases, write down the maximum or minimum turning value, and the corresponding value of x each time.

28. (a) Express $2 + 4x - x^2$ in the form $a - (x + b)^2$.

 (b) Hence write down the coordinates of the minimum turning point on the curve

$$f(x) = \frac{12}{2 + 4x - x^2}.$$

Trig

29. Sketch the graph of :–

 (a) $y = 2\sin 3x°$ $(0 \leq x \leq 360)$

 (b) $y = \cos(x - 60)° + 1$ $(0 \leq x \leq 360)$

30. Solve for $0 \leq x \leq 2\pi$, giving your answer to 2 decimal places when necessary :–

 (a) $\cos^2 x = \frac{1}{4}$ (b) $12\cos^2 x - 5\cos x - 2 = 0$

 (c) $2\sin x + \sqrt{3} = 0$ (d) $2\sin(2x + \frac{\pi}{6}) = 1$.

31. The minimum depth, d feet, of water in a marina, t hours after midnight, can be estimated by the function :–

$$d(t) = 21 + 12\cos(\frac{\pi}{6}t), \text{ where } 0 \leq t \leq 24.$$

 (a) At midnight, a yacht, with a draft of 15 feet, is in the marina.(i.e. it needs a clear 15 ft depth of water to prevent being grounded).
 By what time (24 hour clock) must it leave the marina to prevent being left aground ?

 (b) What is the earliest time after that, the yacht can return to the marina ?

Recurrence Relations

32. A sequence is defined by the recurrence relation :–

$$V_n = 0{\cdot}8V_{n-1} + 3, \qquad V_1 = 4.$$

(a) Calculate the value of V_2

(b) What is the smallest value of n for which $V_n > 11$?

(c) Find the limit of this sequence as $n \longrightarrow \infty$.

33. On the day of her 16th birthday, April is given a sum of money by her uncle to put into the Building Society until she is 21. The money she invests gains compound interest of 7% per annum, which is added on each following birthday.

By what percentage will April's investment have increased when she is allowed to withdraw her money on her 21st birthday ?

34. The first three terms of the linear recurrence relation

$U_{n+1} = aU_n + b$ are 14, 12 and 10 respectively.

Find the values of a and b.

35. Once a week, the head greenkeeper of a golf club removes leaves and slime from the pond at the 12th hole. He generally manages to remove 92% of the debris present at the time.

Each week though, 20 kg of leaves and slime build up in the pond.

The golf club committee have warned him that his job would be in danger if ever the amount of debris reached 22 kg.

Is the greenkeeper's job safe in the long run (explain clearly) ?

36. The adult male population of Wellington is 20000, all of whom are avid supporters of one or other of the two local teams, Rovers and United.

In 1990, it was estimated that half supported Rovers whilst the other half supported United, but they tended to change their loyalty every so often.

The Rovers management estimate that they lose about 20% of their support to United each season. United estimate that they lose 3000 supporters to Rovers each year.

(a) If R_n represents the total Rovers support in season n, show that

$$R_{n+1} = 0{\cdot}8R_n + 3000 \qquad R_0 = 10000$$

(b) What would the Rovers' support be in
(i) 1991 (ii) 1992 (iii) 1993 ?

(c) If this situation were to continue, how many adult fans would each team have in the long run ?

The Circle

37. Show that the equation of the tangent at the point T(1,–3) on the circle $x^2 + y^2 = 10$ is $x - 3y - 10 = 0$.

38. The point (4,–6) lies on the circumference of the circle $x^2 + y^2 - 8x - 16y + c = 0$.
    Find the value of c.

39. Find the equation of the tangent at the point (2,1) on the circle $x^2 + y^2 + 2x - 3y - 6 = 0$.

40. (a) What is the centre and length of radius of the circle $(x - 2)^2 + (y + 1)^2 = 100$.
    (b) A chord AB of this circle has equation $2y = x + 6$.
    Find the coordinates of the points A and B on the circle and write down the coordinates of the mid point of the chord AB.

41. Express the coordinates of the centre and the length of the radius of the circle,

$$x^2 + y^2 - 2px\cos\theta - 2py\sin\theta + p^2\cos^2\theta = 0$$

in terms of p and θ and show that the circle just touches the x–axis.

42. (a) Show that the y - axis is one of the tangents to the circle $x^2 + y^2 - 2x - 10y + 25 = 0$ and find the equation of the other tangent to it through the origin.
    (b) If $y = mx$ cuts this circle in 2 distinct points, find the range of values of m.

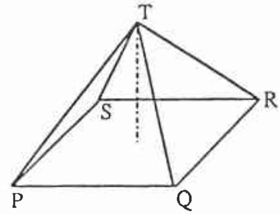
43. Find an expression, in terms of t, for the length of the radius of the circle

$$x^2 + y^2 - (t - 1)x - (t + 1)y + t = 0.$$

44. Find the equation of the circle which passes through A(1,2), B(3,4) and C(7,0).

Trigonometric Formulae

45. PQRST is a pyramid on a square base PQRS and with each of its 8 edges of length 2 units.
    Find the size of the angle between the planes QRT and PQRS.



46. Calculate the exact value of $\sin(x - y)$ if x and y are acute angles, given that $\tan x = \frac{3}{4}$ and $\tan y = \frac{5}{12}$.

47. Solve the equation $3\cos 2x° - 2 = 13\cos x°$ for $0 \leq x < 360$.

48. $P = 4\cos^2\alpha + 2\sin\alpha\cos\alpha + 2\sin^2\alpha$.

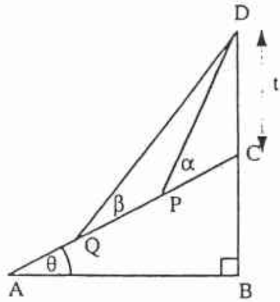
    Express P in the form $K\cos 2\alpha + L\sin 2\alpha + M$ evaluating the constants K, L and M.

49. If $\cos\theta = \frac{12}{13}$, $0 \leq \theta \leq \frac{\pi}{2}$, find the exact value of :–
    (a) $\sin 2\theta$ (b) $\sin 4\theta$.

50. In the diagram $\angle DQP = \beta$ radians
    $\angle DPC = \alpha$ radians
    and length CD is t units.

    (a) Show that $\sin DCA = \cos\theta$ and hence show that
    $$DP = \frac{t\cos\theta}{\sin\alpha}$$
    (b) Find an expression for $\angle PDQ$ in terms of α and β.

    (c) Hence, prove that $PQ = \dfrac{t\cos\theta\sin(\alpha - \beta)}{\sin\alpha\sin\beta}$



Polynomials

51. A(1,0) and B(–2,0) are the two points at which the curve $y = x^4 + 2x^3 - 3x^2 - 4x + 4$ cuts the x–axis. By factorising the expression $x^4 + 2x^3 - 3x^2 - 4x + 4$ fully, prove that there are no other points of intersection with this axis.

52. Find the quotient and remainder when $6x^3 + 7x^2 - x - 2$ is divided by $2x - 1$.

53. Find n if $(x + 3)$ is a factor of $3x^3 + 2x^2 + nx + 6$, and factorise the expression fully when n has this value.

54. A function is defined $g(x) = x^3(3x + 2)$.
    (a) Find the stationary values of g and determine their natures.
    (b) Find where the graph of $g(x)$ cuts the x axis.
    (c) Sketch the graph of $g(x)$.

55. Show that the equation of the tangent to the curve $y = 5 - 2x^2 - x^3$ at $x = -2$ is $y + 4x + 3 = 0$.

56. A function is defined by $p(x) = x^3 + k$ where k is a constant.
    When $p(x)$ is divided by $x - 3$, the remainder is 36.
    Find k and hence solve the equation $p(-2x) = -18$.

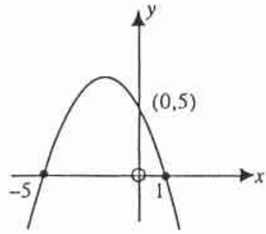
57. (a) Find the stationary points of the function defined by $f(x) = 2 + x^2 - \frac{1}{3}x^3$ and determine their natures.
    (b) Show that the function has a value of zero for a replacement of x between $x = 3$ and $x = 4$ and find this value to one decimal place.
    (c) Sketch the graph of $f(x)$.

Quadratic Theory

58. Find the values(s) of c for which the quadratic equation

$$x^2 - 2x + 21 = 2c(3x - 7)$$

has equal roots.



59. Find the equation of the parabola shown opposite.

60. Find t, given that $x^2 + (t - 3)x = -1$ has no real roots.

61. x is real and $m = \dfrac{x^2 + 4x + 10}{2x + 5}$.
By considering a quadratic equation in x, show that m cannot have a value between –3 and 2.

62. Given that $\dfrac{p}{x} + \dfrac{x+2}{p+1} = 2$, show that $p^2 + p(1 - 2x) + x^2 = 0$.
Hence determine the set of values for x for which p is real.

63. Find the condition for the quadratic equation $(mx + c)^2 = 8x$ to have equal roots.
Hence find the equation of the line through (0,4) which is tangent to the parabola that $y^2 = 8x$.

64. (a) Show that the line $y = 3x + c$ meets the parabola $y = 2x^2 + x - 4$ where

$$2x^2 - 2x + (-4 - c) = 0$$

(b) Find the value of c for the line to be a tangent to the parabola.

(c) Find the point of contact.

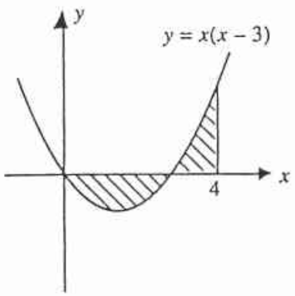
Integration

65\. (a) $\int (\sqrt[4]{x} - 3)\,dx$ (b) $\int \frac{6x^3 - 4x^2 + 2x}{x}\,dx$

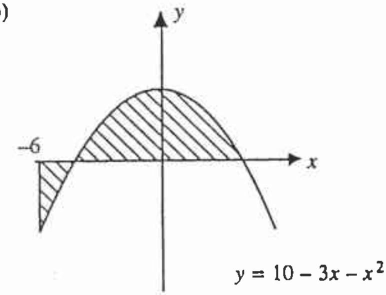
(c) $\int (x^2 - \frac{1}{x^3})\,dx$ (d) $\int_1^3 (\frac{1 + \sqrt{w}}{\sqrt{w}})\,dw$

66\. Calculate the shaded areas :–

(a)



(b)



67\. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 4 - \frac{1}{x^2}$

If the curve passes through the point (1,6), find its equation.

68\. Calculate the area enclosed between the functions

$g(x) = x$ and $h(x) = x^2 - 3x + 3$.

Logarithms and Exponentials

69. $3\log_x 4 - \log_x 8 - \log_x 2 = 1$. Find the value of x.

70. Given $x = \log_3 6 + \log_3 4$, find the value of x, correct to 2 decimal places.

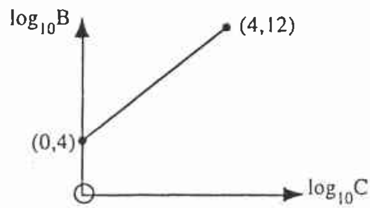
71. Medical students, studying the growth of a strain of bacteria, notice that the number of bacteria present after T hours is given by the formula

$$B(t) = 40e^{1 \cdot 5t}.$$

(a) Write down the number of bacteria present at the start of the experiment.

(b) How many minutes will it take for the bacteria to double in number ?

72. The graph shows a relationship between two variables B and C.
Show there is a formula, connecting B and C of the form $B = kC^n$ and find the values of k and n.



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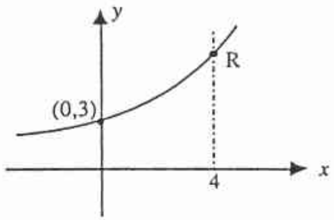


Shown is part of the graph of $Y = Pe^{0 \cdot 3x}$.

(a) Write down the value of P.

(b) Calculate the coordinate of R.

74. A study was made in 1999 to look at the fall in the number of children attending football matches in Scotland.
The formula representing the number of children, is given by

$C(t) = 3000e^{-0 \cdot 2t}$ where

C(t) is the number of children present in year t, and t is the number of years <u>after</u> 1999.

(a) How many children were attending matches at the start of 1999 ?

(b) After how many years will the number of children have fallen by half ?

Further Calculus

75. If $f(x) = \sqrt{1+x^2}$, find $f'(x)$.

76. Given that $f(x) = \cos^2 x$, find $f'(x)$ and then solve the equation

$$f'(x) = \frac{-1}{2} \qquad 0 \leq x < \pi.$$

77. A curve has equation $y = (x+3)^{\frac{1}{2}}$.

Find the equation of the tangent at the point on the curve where $x = 6$.

78. If $f(t) = 2\sin 2t + \cos^3 t$, find $f'(t)$.

79. A function of $f(x) = 1 + \cos(\pi/4 + x)$, where $0 \leq x \leq 2\pi$.

(a) Find the value of x for which $f(x) = 0$

(b) Find the values of x for which $f'(x) = 0$. Hence obtain the stationary values of f in the given interval.

80. For $0 \leq x \leq 2\pi$, find the stationary points on the curve $y = \sin 2x + 2\sin x$.

81. Evaluate :–

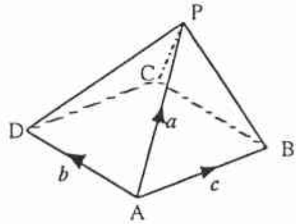
(a) $\int (6x-5)^5 \, dx$ (b) $\int_0^2 \sqrt{4x+1} \, dx$

(c) $\int_1^4 \frac{1}{(3+2x)^2} \, dx$ (d) $\int_0^{\frac{\pi}{3}} (3\sin x + 2\sin 2x) \, dx$.

Further Vectors

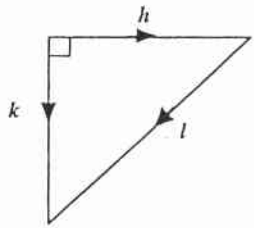
82. K, L and M are the points (4,6,3), (3,1,1) and (5,11,5) respectively.
    (a) Show that K, L and M are collinear.
    (b) Find the coordinates of N such that $\overrightarrow{KN} = 3\overrightarrow{KL}$.

83. In this square based pyramid, all 8 edges have length 2 units.
    $\overrightarrow{AP} = \boldsymbol{a}$, $\overrightarrow{AD} = \boldsymbol{b}$ and $\overrightarrow{AB} = \boldsymbol{c}$.
    Evaluate $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c})$.



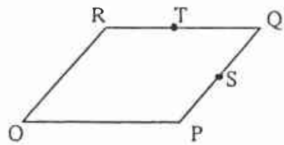
84. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows :–
    $\boldsymbol{p} = 4\boldsymbol{i} - 2\boldsymbol{k}$; $\boldsymbol{q} = 2\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$; $\boldsymbol{r} = 2\boldsymbol{j} + 2\boldsymbol{k}$.
    (a) Evaluate $\boldsymbol{p}.\boldsymbol{q} + \boldsymbol{p}.\boldsymbol{r}$
    (b) From your answer, say something about the vector $(\boldsymbol{q} + \boldsymbol{r})$.

85. Shown opposite is a right angled isosceles triangle whose sides represent the vectors $\boldsymbol{h}$, $\boldsymbol{k}$ and $\boldsymbol{l}$.
    The two equal sides have length 4 units.
    Find the value of $\boldsymbol{k}.(\boldsymbol{h} + \boldsymbol{k} + \boldsymbol{l})$.



86. Calculate the angle between between the two vectors,
    $\boldsymbol{p} = \boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$.

87. OPQR is a parallelogram. S is the mid point of PQ and T the midpoint of QR. Relative to the origin O, $\boldsymbol{p}$, $\boldsymbol{q}$, $\boldsymbol{r}$, $\boldsymbol{s}$ and $\boldsymbol{t}$ are the position vectors of P, Q, R, S and T respectively.
    (a) Express $\boldsymbol{q}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$.
    (b) Express $\boldsymbol{t}$ in terms of $\boldsymbol{q}$ and $\boldsymbol{r}$.
    (c) Hence, show that $4(\boldsymbol{s} + \boldsymbol{t}) = 3(\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$.



88. Find the coordinates of M which divides K(2,1,3) and N(6,5,11) in the ratio 3:1.

89. P, Q and R are the points (0,5,5), (4,1,1) and ($2\frac{1}{2}$,$2\frac{1}{2}$,$2\frac{1}{2}$) respectively.
    (a) Prove that P, Q and R are collinear and find the ratio which R divides PQ.
    (b) If O is the origin, prove that OR bisects angle POQ.

Wave Function

90. (a) Express $4\sin x° - 8\cos x°$ in the form $R\sin(x - \alpha)°$ where $R > 0$ and $0 \le \alpha < 360$.

(b) Find the maximum value of $f(x) = 4\sin x° - 8\cos x°$ and state the value of x at which this maximum occurs.

91. Express $\sqrt{3}\cos x° + \sin x°$ in the form $k\cos(x - \alpha)°$ and hence solve the equation :–

$$\sqrt{3}\cos x° + \sin x° = 1{\cdot}6 \qquad 0 \le x \le 360.$$

92. Show that $3\cos x° - 2\sin x°$ can be expressed as $\sqrt{13}\cos(x + 33{\cdot}7)°$ and hence solve the equation :–

$$3\cos x° - 2\sin x° = 1 \text{ for } 0 \le x < 360.$$

93. $E = 4\cos^2\theta + 2\sin\theta\cos\theta + 2\sin^2\theta$.

(a) Express E in the form $A + B\cos 2\theta + C\sin 2\theta$ evaluating the constants A, B and C.

(b) Hence express E in the form $p + q\cos(2\theta - \alpha)$, evaluating p, q and α.

(c) State the maximum and minimum values of E.

94. Find the maximum value of $20\cos 4\beta + 20\sin 4\beta$ where $0 \le \beta \le \pi$ and state both values of β at which this maximum occurs.

95. The expression $25\sin 10t° + 50\cos 10t°$ represents a displacement of a wave after t seconds. This expression can be written in the form $A\sin(10t + \alpha)°$ where $A > 0$ and $0 \le \alpha \le 360$.

(a) Find the values of A and α.

(b) Write down the amplitude of the wave.

(c) Use your values of A and α to sketch the graph of $A\sin(10t + \alpha)°$ against t for $0 \le \alpha \le 360$ showing clearly the points where the graph cuts the t axis and any stationary points.

Answers to Revision Section

Equations of Lines

1. (a) $y = 3x + 8$ (b) $y = {}^1/_2x + 7$ (c) $y = 3x + 12$.

2. $3y + 4x = 1$.

3. $y - 1 = {}^{-3}/_2(x - 5)$.

4. $y - 5 = -2(x + 3)$.

5. (a) $3y - 2x = -14$ (b) $y + {}^3/_2x = 17$, (10,2).

6. (a) $y = 2x + 3$ (b) Proof.

7. (a) $y = 2x - 10$ and $y = {}^1/_4x + {}^1/_2$ (b) $({}^{14}/_3, {}^{-2}/_3)$.

8. −1·2.

9. 0·29.

Differentiation

10. (a) $20x^3 - 3$ (b) $12x + \frac{3}{x^2}$ (c) $\frac{-18}{x^3} + \frac{3}{2}x^{\frac{1}{2}}$
(d) $\frac{9}{2}(x^{\frac{1}{2}} - x^{-\frac{1}{2}})$ (e) $3 - \frac{5}{2x^2}$.

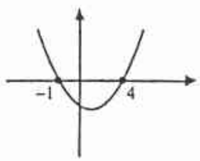
11. $\frac{5}{128}$.

12. (a) 36 m/s (b) 0 m/s; top of arc, stationary.

13. (a) $y + 3 = -(x - 1)$ (b) 2 and −1.

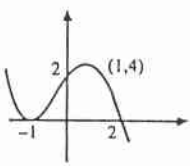
14. 6, 35·5°.

15.

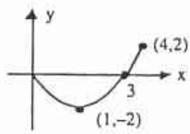


16. 32, 0.

17. 4 & 0



18. (a) $\frac{3}{2}\sqrt{x} - \frac{3}{2\sqrt{x}}$ (b) (4,2) $(y - 2) = \frac{9}{4}(x - 4)$ (c)



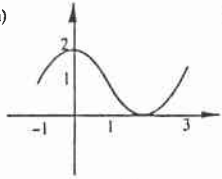
Graphs/Functions

19. 2.

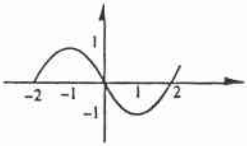
20. 2p.

21. (a) Not in 1 to 1 correspondence (b) $\frac{2}{1-x}$ $x \in R$; $x \neq 1$

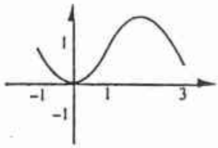
22. (a)



(b)



(c)



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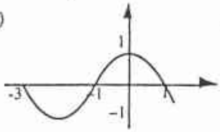

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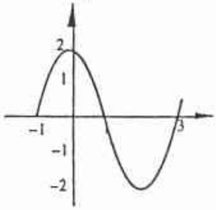
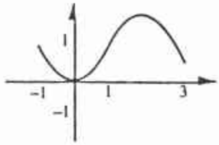


(f)



23. a = 2, b = 2.

24. c = 3.

25. (a) $1 - 2\sin^2 x°$; $^1/_2$. (b) $f^{-1}(x) = \frac{1-x}{2}$; p, q – no inverses.

Completing the Square

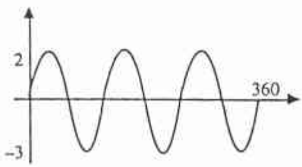
26. (a) $(x + 3)^2 - 9$ (b) $(x + 5)^2 - 24$
   (c) $9 - (x - 2)^2$ (d) $2(x + {}^3/_2)^2 - 3^1/_2$
   (e) $7 - 3(x - 1)^2$.

27. (a) (–3,–9) (b) (–5,–24) (c) (2,9) (d) $(-^3/_2, -3^1/_2)$ (e) (1,7).

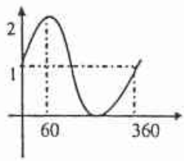
28. (a) $6 - (x - 2)^2$ (b) minimum at (2,2).

Trig

29. (a)



(b)



30. (a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (b) 0·84, 1·82, 4·46, 5·44 radians
(c) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (d) $0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$.

31. (a) 0400 (b) 0800.

Recurrence Relations

32. (a) 6·2 (b) 6 (c) 15.

33. 40%.

34. a = 1, b = –2.

35. YES it limits to about 21·7 kg.

36. (a) Proof (b) (i) 11000 (ii) 11800 (iii) 12440 (c) Rovers = 15000, United = 5000.

The Circle

37. Proof.

38. c = –116.

39. y = 6x – 11.

40. (a) (2,–1), 10. (b) (–8,–1) (8,7), mid point (0,3).

41. Centre (pcosα, psinα) radius = psinaα; Proof.

42. (a) $y = \frac{12}{5}x$ (b) $m > \frac{12}{5}$.

43. $r = \sqrt{\frac{t-1}{\sqrt{2}}}$.

44. $x^2 + y^2 - 8x - 2y + 7 = 0$.

Trigonometric Formulae

45. 54·8° (approx.)

46. $\frac{16}{65}$.

47. 109·5 and 250·5.

48. K = 1, L = 1, M = 3.

49 (a) $\frac{120}{169}$ (b) $\frac{28560}{28561}$.

50. (a) Proof by sine rule (b) $\angle PDQ = \alpha - \beta$ (c) Proof by sine rule.

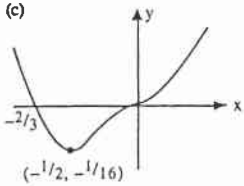
Polynomials

51. Proof showing 1 and –2 appear again as factors.

52. Quotient $3x^2 + 5x + 2$, remainder 0.

53. $n = -19$, $(x + 3)(3x - 1)(x - 2)$.

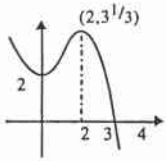
54. (a) (0,0) is a point of inflection $(-\frac{1}{2}, -\frac{1}{16})$ is a minimum turning point
(b) cuts x-axis (0,0) $(-\frac{2}{3},0)$
(c)



55. Proof.

56. $k = 9$, $x = \frac{3}{2}$.

57. (a) Min (0,2), Max $(2,3\frac{1}{3})$ (b) Proof, 3·5. (c)



Quadratic Theory

58. $c = 2$ or $-\frac{10}{9}$.

59. $y = 5 - 4x - x^2$.

60. $(1 < t < 5)$.

61. Proof.

62. $(x \leq \frac{1}{4})$.

63. $(mc = 2)$, $y = \frac{1}{2}x + 4$.

64 (a) Proof (b) $c = \frac{-9}{2}$ (c) $(\frac{1}{2},-3)$.

Integration

65. (a) $\frac{4}{5}x^{\frac{5}{4}} - 3x + c$ (b) $2x^{1} - 2x^{2} + 2x + c$

(c) $\frac{1}{3}x^{3} + \frac{1}{2x^{2}} + c$ (d) $2\sqrt{3}$.

66. (a) $6^{1}/3$ units^2 (b) 61 units^2.

67. $y = 4x + \frac{1}{x} + 1$.

68. $1\frac{1}{3}$ units^2.

Logarithms and Exponentials

69. 4.

70. 2·89.

71. (a) 40 (b) 27·7 minutes.

72. $B = 10000c^2$.

73. (a) 3 (b) (4,9·96).

74. (a) 3000 children (b) $3\frac{1}{2}$ years.

Further Calculus

75. $\frac{x}{\sqrt{1+x^2}}$.

76. $-\sin 2x$, $\frac{\pi}{12}, \frac{5\pi}{12}$

77. $y = \frac{1}{6}x + 2$.

78. $4\cos 2t - 3\cos^2 t \sin t$.

79. (a) $\frac{3\pi}{4}$ (b) $\frac{3\pi}{4}, \frac{7\pi}{4}$ 0 and 2.

80. Sketch showing $(\frac{\pi}{3}, \frac{3\sqrt{3}}{2})$ Max, $(\pi,0)$ Inflection , $(\frac{5\pi}{3}, \frac{-3\sqrt{3}}{2})$ Min.

81. (a) $\frac{(6x-5)^6}{36} + c$ (b) $4\frac{1}{3}$ (c) $\frac{3}{55}$ (d) 3.

Further Vectors

82. (a) Proof (b) $(1,-9,-3)$

83. 4.

84. (a) 0 (b) Perpendicular.

85 32.

86 74°.

87. (a) $p + r$ (b) $\frac{1}{2}q + \frac{1}{2}r$ (c) Proof.

88. M(5,4,9).

89. Proof 5:3.

Wave Function

90 (a) $4\sqrt{5}\sin(x-63\cdot4)°$ (b) Max $4\sqrt{5}$ at $x = 153\cdot4$.

91. $2\cos(x-30)°$ $x = 66\cdot9, 353\cdot1$.

92. Proof $x = 40\cdot2$ or $252\cdot4$.

93. (a) $A = 3, B = 1, C = 1$ (b) $p = 3, q = \sqrt{2}, \alpha = 45$ (c) $3 + \sqrt{2}, 3 - \sqrt{2}$.

94. $20\sqrt{2}$ $\frac{\pi}{16}, \frac{9\pi}{16}$.

95. (a) $A = 55\cdot9, \alpha = 63\cdot4$ (b) $55\cdot9$ (c) Sketch.